Wedding Options

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1 What this paper pursues.

The purposes of this paper are:
1. To show to my friends, who are non-financial mathematicians, that financial mathematics is not only exiting but also practical.
2. To demonstrate to the students, studying quantitative finance the application [and limits] of the models they learn.
3. To amuse expirienced quants.

Additionally, the problem I formulate is equivalent[1] to that of a portfolio manager, who made losses and is in danger to be dismissed. In this case his [selfish] objective is not to continue trying to maximize the returns under minimal risk but rather to win back what he has lost, disregarding the risk of further losses.

2 Introduction

What are the wedding options? Are they yet another type of exotic derivatives?! No, they are just plain-vanilla call options, in which we are going to invest in order to finance our wedding.

It is well known that the girls, at least the Russian girls, burn with the desire to have a magnificent wedding... and how costly this wedding can be. Olesja estimates the wedding budget at 5000€ but I find it prohibitive. I, being a Sagittarius, also make no secret of the fact that I consider all wedding expenditures as a waste. But making Olesja happy is not a waste, so if a splendid wedding makes her happy, I am going to discuss it. However, 5000€ find I too much, 2000€ is OK but no more. And since family happiness does not mean to satisfy each others whim but does mean to live in concord, we searched for a solution, which suits us both. A straightforward idea is to try cutting the costs downto 2000€ but this would be an excellent example for a Parkinson's Law, which states that a compromise is often worse than the both alternatives. Indeed, such an emasculated wedding would neither make Olesja happy, nor satisfy me since 2000€ is still not a Taschengeld(pocket money).

So we found a non trivial solution: I invest money on financial market and if I make profit we WILL have a magnificent wedding else we will do without it. But to make 5000€ from 2000€ is an ambitious challenge. It IS realistic, since we do not plan to marry tomorrow but rather in two or three years. However, the stocks are not the best assert to achieve the required

[1]It was Stephan Vorgrimler, who pointed me at this analogy.

return of 150% since the stock prices have already significantly grown from the March minima. So if not stocks, than what? Forex, CFD, exotic leveraged products? No, these things are TOO risky, they might have been an ultima ratio if we were to marry tomorrow. But since we have some time, I prefer the call options on stocks, which are liquid, transparent and cannot cause the losses larger than the initial investment.

3 Making a choice: options on which stocks?

Because our investment is going to be relatively long-term it must be the stocks of well-established companies. So the obvious idea is to have a look at DAX itself and its components[2]. To choose the options I will use an excellent comdirect.de database/option selector.

There are 30 companies in DAX but only for 14 of them there are call options with maturity in 3 years or more. These are the following companies:

Allianz	Deutsche Telecom
Basf	E.ON
Bayer	Munich Re
Daimler	RWE
Deutsche Bank	SAP
Deutsche Lufthansa	Siemens
Deutsche Post	Volkswagen

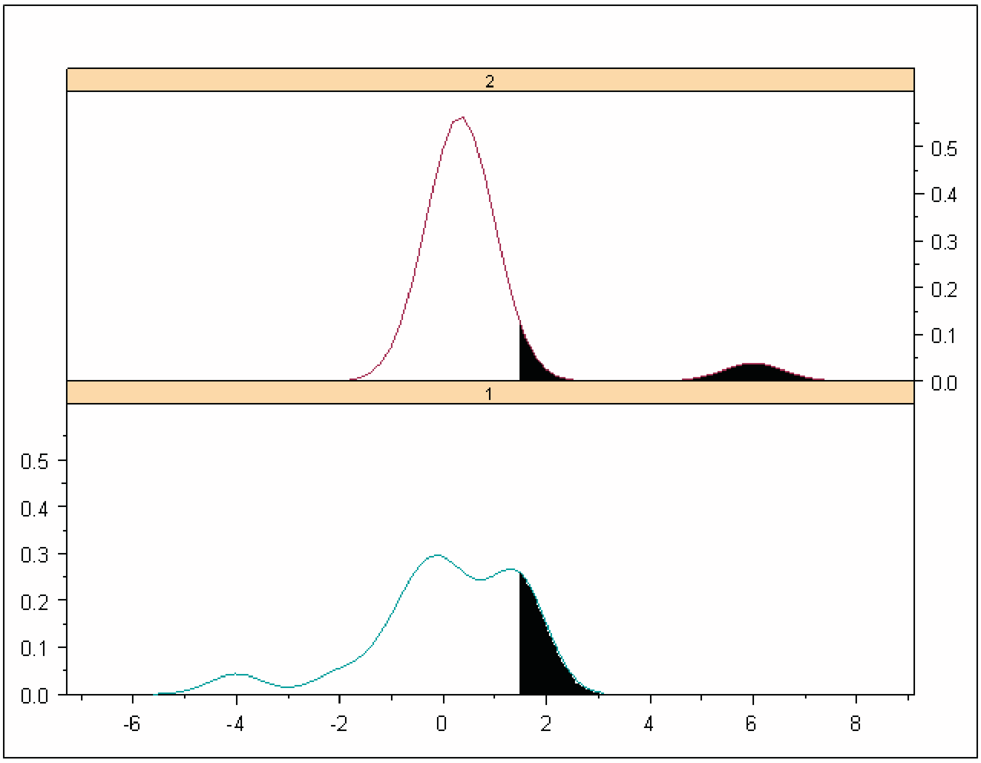
As to portfolio modeling, fifteen entities (these 14 plus DAX itself) is enough to make the simulation complicated[3] and insufficient for portfolio homogeneity. The easiest solution is obviously to select only one, the most promising stock and then find an optimal option [combination] on it. That is exactly what I will do [as a first step]. In this univariate case I get rid of "curse of correlation"[4] and can easily visualize the results.

Once again, in context of my goal I do not maximize the risk adjusted performance, I maximize the probability that the profit will be 150% or more, disregading the risk. To explain it graphically: suppose there are two assets with the following profit/loss probability densities.

[2]There is also a big liquidity advantage, esp. concerning the Bid/Ask spread

[3]Do not forget that for each stock there are many options with different strikes and maturities.

[4]I mean, e.g. that even if we thoroughly measure the historical correlation, it still can be different in future.



Though the 2nd has larger expectation and stronger positive skewness, I choose the 1st one since it has probability mass within $[1.5, \infty)$ (solid black area).

So I start with a classical lognormal model and assume that the stock price evolves according to the following SDE

$$dS_t = S_t(\mu dt + \sigma dW_t) \tag{1}$$

or equivalently

$$S_t = S_0 e^{(\mu - \sigma^2/2)t + \sigma W_t} \tag{2}$$

As is well known, the option value can be decomposed to the sum of the intrinsic value $[S - K]^+$ (what we get if exercise an option) and the time value(which captures the possibility that the option value may increase due to volatility of underlying stock).
For simplicity I [so far] disregard the time value and model only the intrinsic value of an option, which is (at time t) completely determined by stock price S_t. So I initially invest 2000€ in American call options and I am going to execute them as soon as the stock price is high enough to get 5000€ , i.e. as soon as $S_t - K = 2.5C_0$ (as usually K means strike and C_0 is initial option price). Further I denote the price, which satisfies this equality with S

Wedding.
If the price stays below S Wedding I still keep my options and exectute them on maturity date[5].
In order to estimate the daily drift μ and volatility σ for the model I download the historical stock prices[6] from de.finance.yahoo.com and transform the time series of the stock prices S_t to the time series of returns r_t

$$r_t = \frac{S_t - S_{t-1}}{S_{t-1}} \tag{3}$$

Then[7] the sequence $\{r_t\}$ consists of i.i.d. $N(\mu, \sigma^2)$ random variables.

With estimated μ and σ I model for every stock 100000 scenarios of the price evolution according to eq. (2).
Finally, for each stock there are several options C (with different strikes K and maturity dates T) on the market. However, there is only one option issuer, which offers long-term options under reasonable conditions, it is HSBC Trinkaus & Burkhardt AG[8]. I choose the options with the latest maturity, which for the most of options is 13.12.2013 and for each of them estimate:

1. the probability P_w that I yield 5000€ .
2. the probability P_p that I either yield 5000€ or at least do not suffer a loss.
3. the probability P_{be} that I suffer no loss if would just keep the options to maturity date.

P_{be} is used just to verify the simulation results since its value can be calculated analitically $P_{be} = 1 - \mathbb{P}\{S_T \leq S \text{ breakeven}\}$, where S_T is

$$LogNormally\left(\ln(S_0) + (\mu - \sigma^2/2)\#(T), \sigma\sqrt{\#(T)}\right)$$

distributed, $\#(T)$ means the number of business days before the maturity date and S breakeven $= C_0 + K$.

[5]Of course only if their (intrinsic) value on maturity date is still positiv.

[6]I consider the prices on Frankfurt Stock Exchange, not in electronic trading system XETRA, since for XETRA the data are available only from 2003.

[7]Strictly speaking, I should have taken $ln(S_t/S_{t-1})$, thanks to Alexey Ivanov for this note. But the numerical difference between simple and logariphmic returns is subtle. In both cases Q-Q plot shows an acceptable fitting to the normal distribution with some discrepancy in tails.

[8]There are a few long term options from WestLB and Goldman Sachs but for the latter the Bid/Ask spread is prohibitive.

The simulation yields the following results:

Allianz. $T = 13.12.2013$, $S_0 = 63.28, \mu = 0.000977, \sigma = 0.0253958$							
WKN	K	$C(Ask)$	S breakeven	S Wedding	P_w	P_p	P_{be}
TB2RE1	100.00	11.00	111.00	127.50	0.71	0.72	0.59
TB2RE2	120.00	7.70	127.70	139.25	0.66	0.67	0.53
TB2VGP	76.00	17.00	93.00	118.50	0.75	0.77	0.67

BASF. $T = 13.12.2013$, $S_0 = 29.11, \mu = 0.000572, \sigma = 0.029599$							
WKN	K	$C(Ask)$	S breakeven	S Wedding	P_w	P_p	P_{be}
TB2RE4	40.00	3.40	43.4	48.5	0.65	0.65	0.40
TB2RE5	32.00	5.40	37.4	45.5	0.69	0.70	0.46

BAYER. $T = 13.12.2013$, $S_0 = 38.72, \mu = 0.000216, \sigma = 0.021947$							
WKN	K	$C(Ask)$	S breakeven	S Wedding	P_w	P_p	P_{be}
TB2REB	68.00	2.50	70.50	74.25	0.36	0.36	0.20
TB2REA	52.00	4.90	56.90	64.25	0.47	0.48	0.29

Daimler. $T = 13.12.2013$, $S_0 = 24.98, \mu = 0.000311, \sigma = 0.022437$							
WKN	K	$C(Ask)$	S breakeven	S Wedding	P_w	P_p	P_{be}
TB2VH8	24.00	9.30	33.30	47.25	0.43	0.48	0.39
TB2RF9	32.00	6.50	38.50	48.25	0.40	0.43	0.32
TB2RFA	40.00	4.50	44.50	51.25	0.37	0.38	0.25

Deutsche Bank. $T = 13.12.2013$, $S_0 = 41.97, \mu = 0.0000825, \sigma = 0.0267784$							
WKN	K	$C(Ask)$	S breakeven	S Wedding	P_w	P_p	P_{be}
TB2S5C	28.00	20.70	48.70	79.75	0.36	0.41	0.30
TB2RF6	32.00	18.70	50.70	78.75	0.37	0.41	0.29
TB2RF7	40.00	15.20	55.20	78.00	0.37	0.40	0.26
TB2ZVT	52.00	11.10	63.10	79.75	0.36	0.37	0.21
TB235B	68.00	6.80	74.80	85.00	0.32	0.32	0.16

Lufthansa. $T = 12.2.2013$, $S_0 = 24.98, \mu = 0.000193, \sigma = 0.019476$							
WKN	K	$C(Ask)$	S breakeven	S Wedding	P_w	P_p	P_{be}
WLB9ES	0.50	7.10	7.60	18.25	0.21	0.60	0.59

E.ON. $T = 13.12.2013$, $S_0 = 24.14, \mu = 0.000812, \sigma = 0.044813$							
WKN	K	$C(Ask)$	S breakeven	S Wedding	P_w	P_p	P_{be}
TB2VHG	28.00	3.60	31.60	37.00	0.73	0.73	0.37
TB2U86	32.00	2.70	34.70	38.75	0.71	0.71	0.35
TB2RG4	36.00	1.90	37.90	40.75	0.68	0.68	0.33
TB2RG5	44.00	1.10	45.10	46.75	0.61	0.61	0.29

Munich Re. $T = 13.12.2013$, $S_0 = 95.25, \mu = 0.00030, \sigma = 0.020652$							
WKN	K	$C(Ask)$	S breakeven	S Wedding	P_w	P_p	P_{be}
TB2VJ5	140.00	11.00	151.00	167.50	0.46	0.47	0.30
TB2RHB	160.00	7.30	167.30	174.60	0.43	0.43	0.25
TB2RHC	200.00	4.20	204.20	210.50	0.29	0.30	0.17

Munich Re. $T = 19.06.2013$, $S_0 = 95.25, \mu = 0.00030, \sigma = 0.020652$							
WKN	K	$C(Ask)$	S breakeven	S Wedding	P_w	P_p	P_{be}
GS1P4Q	80.00	29.50	109.50	153.75	0.50	0.57	0.47
GS1P4Y	100.00	20.60	120.60	151.50	0.51	0.55	0.41
GS1P56	120.00	15.00	135.00	157.50	0.48	0.50	0.35

RWE. $T = 13.12.2013$, $S_0 = 55.27, \mu = 0.000646, \sigma = 0.01771$							
WKN	K	$C(Ask)$	S breakeven	S Wedding	P_w	P_p	P_{be}
TB2VJK	68.00	4.50	72.50	79.25	0.82	0.83	0.69
TB2RHJ	76.00	3.20	79.20	84.00	0.78	0.78	0.64
TB2RHH	92.00	1.70	93.70	96.25	0.68	0.68	0.53

SAP. $T = 13.12.2013$, $S_0 = 27.84, \mu = 0.000434, \sigma = 0.019611$							
WKN	K	$C(Ask)$	S breakeven	S Wedding	P_w	P_p	P_{be}
TB2RHN	32.00	6.40	38.40	48.00	0.55	0.59	0.48
TB2RHM	40.00	4.00	44.00	50.00	0.52	0.53	0.39

Siemens. $T = 13.12.2013$, $S_0 = 47.37, \mu = 0.000214, \sigma = 0.023822$							
WKN	K	$C(Ask)$	S breakeven	S Wedding	P_w	P_p	P_{be}
TB2VJP	60.00	8.80	70.80	82.00	0.45	0.46	0.27
TB2S6B	68.00	6.60	74.60	84.50	0.43	0.44	0.25
TB2RHR	76.00	4.90	80.90	88.25	0.40	0.40	0.22
TB2RHS	84.00	3.70	87.70	93.25	0.36	0.36	0.19

I do not consider option on Deutsche Telecom since I strongly believe it cannot withstand mobile competitors anymore[9]. The long term option on Deutsche Post(actually, there is only one, WLB9ER) is unfortunatelly illiquid. Finally, I keep out of options on Volkswagen, because it is an area, where big Porsche guys play their game... and I do remember the sorrowful end of Adolf Merckle, who committed a suicide after very unsucessful speculation with Volkswagen Stocks.

Looking at the results one can see some peculiarities:

1. If all options are out of the money, it is optimal to buy the one with the smallest strike K. If my goal were to maximize the risk-adjusted profit, it likely would not be the case. But I ignore the risk of losses and maximize not the profit itself but the probability that the profit exceeds 150%.

2. The options, which are in the money are expensive and in this case it can be unoptimal to buy an option with the smallest strike; take a closer look at Deutsche Bank.

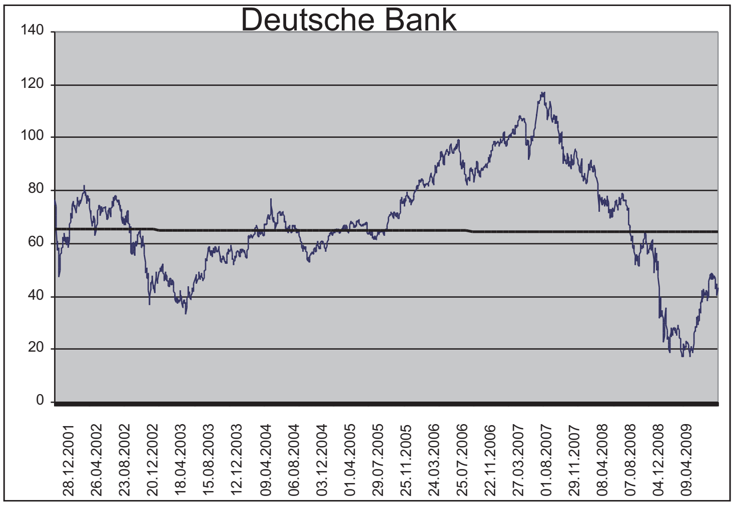
3. (A special remark for students). As you probably learnt in financial math, on the arbitrage free market the equality $(\mu_1 - r)/\sigma_1 = (\mu_2 - r)/\sigma_2$ (r stands for risk free interest rate) must hold for all assets. It is interpreted as the market price of risk. Looking at e.g. RWE vs. Siemens we see that this ratio does not hold.
   You must also have taken a course in empirical finance, like ass@ pricing, where $(\mu - r)/\sigma$ is known as the *Sharpe ratio* and differs from asset to asset. The higher it is, the more attractive is an asset.
   Well, what we learn in financial mathematics is [mostly] based on model assumptions and if they are wrong[10], either the following conclusions may be.
   However, empirical finance are not viceless too, since the measurement of market parameters is error prone. And empirical guys like statistics but strongly dislike to check whether the tools they use are applicable in context of their problems(key phrase: linear regression and t-test forever).

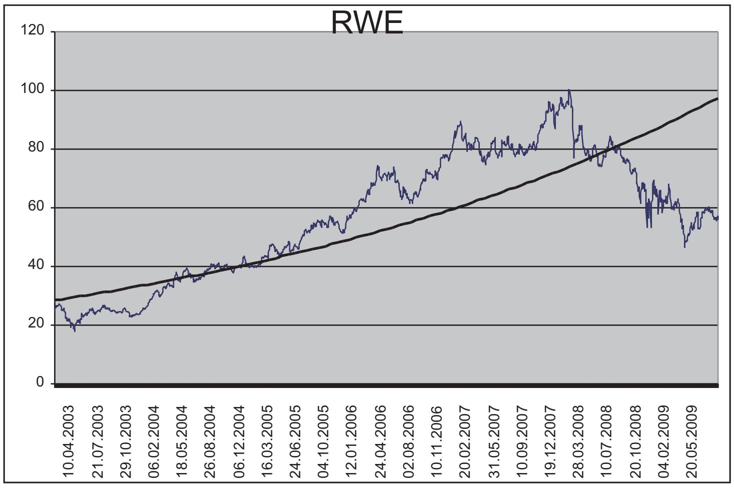
So the results tell us that the best strategy would be to buy a TB2VJK option on RWE. But let us not forget the results come from a model, actually

[9] An will confront a collapse, maybe in near future.

[10] There are some attempts to quantify the *model risk*, i.e. probable losses caused by model misspecification.

a simplistic model.
So instead of blindly believing in the model, we have a look at stock price charts.





We see that Deutsche Bank shares have recently dropped sixfold, whereas RWE shares only twofold. That is, in part, the cause why the drift μ is so small for Deutsche Bank.
But empirical evidences tell us that if a stock falls particularly strong (but the company does not crash), it will likely grow intensively in future. Additionally, we see that from historical point of view the shares of Deutsche Bank were at the bottom[11], whereas RWE stocks do have space to fall further...

And actually these simple considerations convince me more, so it were

[11] However, one used to say that the stock prices reached the bottom ... and began to dig.

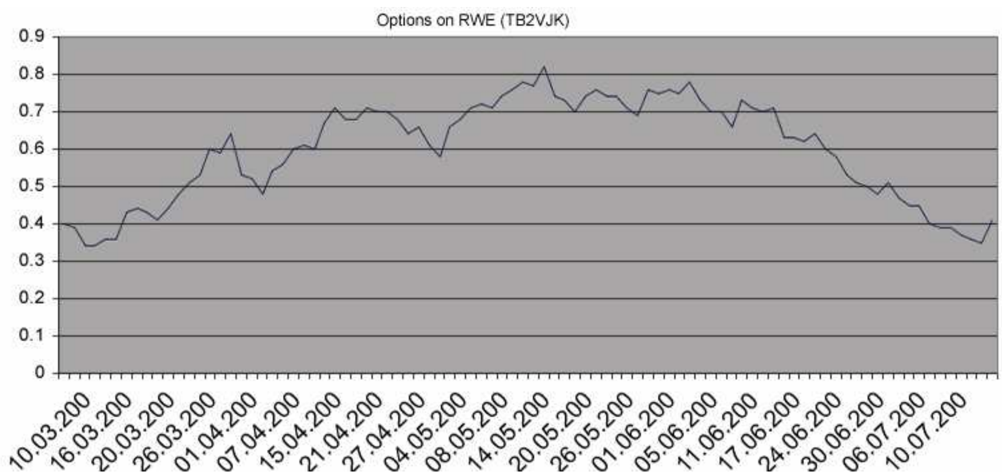
options on Deutsche Bank (TB2RF7) what I bought. So the die is cast! But post factum it would be interesting (from theoretical point of view) to do the following analysis:

1. Try to simulate the time value of options as well. Consider a model with jumps and stochastic volatility, e.g. glorified GARCH(1,1). Stochastic interest rate and dividends taken into account are worthwhile too.

2. Additionally, consider the multivariate case, introducing some "advanced" co-dependence, e.g. copula.

3. Finally, consider the dynamic portfolio management, i.e. not just purchase once and wait but adjust the portfolio as long as market unfolds. Mathematically it would be especially exciting to find a closed-form solution under assumption of continuous trading. Closer to reality is to write a computationally-efficient software, which can calculate the portfolio strategy in real time. Overnight computation/daily adjustment is much easier to implement and still will do, since intraday portfolio adjustment is costly due to transaction fees.

Update: 16.07.2009
As I said, I itinially wanted to buy options once and hold them. But... it contradicts to my dynamic nature. So on 14.07.2009 the shares of Deutsche Bank jumped hoch, whereas the growth of RWE was moderate. Additionally, the options on RWE were again relatively cheap.



So I sold options on Deutsche Bank(with profit 3%) and bought those on RWE(as the modeling results advice). Post factum one can see that it was a good idea, since [so far] RWE grows, Deutsche Bank falls.

On the other hand I dislike such dabbling (keyword: trader's discipline). Thus I do have a good stimulus to develop a dynamical model.

Update: 2.08.2009
On 28.07.2009 Deutsche Bank dropped more than 10%, so I sold options on RWE (with 25% profit) and got again into TB2RF7 at price of 19.01€ . So far it is not very good, since Deutsche Bank continue falling although at a much slower pace. Nevertheless I feel safer, because there were already some similar patterns(K+S, later Merck and now Deutsche Bank): at first rapid growth and that jump down. But one more jump down is unlikely (yes, I know that Poisson process is memoryless :)).